sequently quantified by calibration against Gardner's curves. Table 1 shows these values of simulated λ for various samples of laminate. These values were then used in the simulation of square and rectangular fins. Derived values of fin efficiency are shown compared with numerical results in Fig. 3.

Table 1. Derived values of λ for various samples of laminate

Sample No.	No. of paper thicknesses	Derived λ (m^{-1})	λ mean (m^{-1})
1	3	49·606	49·675
2	3	48·819	
3	3	49·600	
4	4	42·323	42·717
5	4	42·520	
6	4	43·307	
7	4	42·520	
8	4	42·913	
9	5	34·646	34·252
10	5	33·858	

CONCLUSIONS

The accuracy obtainable with this analog is determined by the error in the calibrated value of λ. This can, with care, be kept within 10 per cent, and by varying the number of paper thicknesses in the lamination, a wide range of λ may be simulated. A comparison of analytical and numerical data with analog results shows a deviation of less than ± 7 per cent, and this technique has been used successfully in the evaluation of sheet fins which have not been amenable to analytical or numerical solution.

REFERENCES

1. D. Vitkovitch, *Field Analysis*, Chapters 1, 2 and 5. D. Van Nostrand, London (1966).
2. K. A. Gardner, Efficiency of extended surface, *Trans. Am. Soc. Mech. Engrs* 621 (1945).
3. T. E. Schmidt, Heat transfer calculations for extended surface, *Refrigng Engng* 351–357 (April 1949).
4. H. Zabronsky, Temperature distribution and efficiency of a heat exchanger using square fins on round tubes, *Trans. Am. Soc. Mech. Engrs* **22E**, 119–122 (1955).
5. D. G. Rich, The efficiency and thermal resistance of annular and rectangular fins, in *Third International Heat Transfer Conference*, Vol. 3, p. 281 (1966).

Int. J. Heat Mass Transfer. Vol. 18, pp. 344–345. Pergamon Press 1975. Printed in Great Britain

ON THE SOLUTION OF THE HEAT EQUATION WITH TIME DEPENDENT COEFFICIENT

M. D. MIKHAILOV
Applied Mathematics Centre, VMEI, Sofia, Bulgaria

(*Received* 30 *June* 1974)

NOMENCLATURE

$A(N,\tau)$, $B(N,\tau)$, $\varphi(N,\tau)$, time dependent boundary functions defined on S;
$f_0(M)$, initial distribution in V;
$k(M,\tau)$, $w(M,\tau)$, $P(M,\tau)$, $\rho(M,\tau)$, prescribed functions defined in V;
M, point in V;
N, point on S;
n, outward normal of S;
$T(M,\tau)$, unsteady potential distribution defined in equation (1), (2) and (3);
$\beta(\tau)$, $\gamma(\tau)$, prescribed function defined in τ;
τ, time variable;
$\psi_i(M,\tau)$, eigenfunctions;
$\mu_i(\tau)$, eigenvalues.

IN A RECENT paper [1], the author presented an analytical solution for a large class of heat-transfer problems. The present communication complements [1] applying the method of finite integral transforms for the solution of a more general mathematical model of transfer process with time and space dependent parameters.

Consider the following boundary value problem in a finite homogeneous region of arbitrary geometry V

$$\gamma(\tau)w(M,\tau)\frac{\partial T(M,\tau)}{\partial \tau} = \operatorname{div}[k(M,\tau)\operatorname{grad}T(M,\tau)] + [\beta(\tau)w(M,\tau)-\rho(M,\tau)]T(M,\tau)+P(M,\tau), \quad M\in V, \quad \tau \geqslant 0 \quad (1)$$

subject to the initial condition

$$T(M,0) = f_0(M) \quad (2)$$

and the boundary conditions

$$A(N,\tau)\frac{\partial T(N,\tau)}{\partial n} + B(N,\tau)T(N,\tau) = \varphi(N,\tau). \quad (3)$$

In [1] is solved the particular case where $w(M,\tau)$, $k(M,\tau)$, $\rho(M,\tau)$, $A(N,\tau)$ and $B(N,\tau)$ are not functions of the time τ.

It is supposed that the solution of the problem can be represented in the form of an eigenfunction expansion, with the assumption that the eingenvalue problem

$$\operatorname{div}[k(M,\tau)\operatorname{grad}\psi_i(M,\tau)] + [\mu_i^2(\tau)w(M,\tau)-\rho(M,\tau)]\psi_i(M,\tau) = 0 \quad (4)$$

subject to the boundary condition

$$A(N,\tau)\frac{\partial\psi_i(N,\tau)}{\partial n}+B(N,\tau)\psi_i(N,\tau)=0 \quad (5)$$

is granted for known.

Equations (4)–(5) do not belong to the conventional Sturm–Liouville family because the eigenfunctions $\psi_i(M,\tau)$ and eigenvalues $\mu_i(\tau)$ depend on τ. The idea of time dependent eigenvalues we borrow from Vidin, who presented in [2] an approximate method of temperature field calculations in an infinite cylinder that the convective heat-transfer coefficient is an arbitrary time function.

To solve equation (1) at the conditions (2) and (3), the finite integral transform

$$\bar{T}_i(\tau)=\int_V w(M,\tau)\psi_i(M,\tau)T(M,\tau)\,\mathrm{d}V \quad (6)$$

is to be used.

It follows, from the orthogonality of eigenfunctions $\psi_i(M,\tau)$, that $T(M,\tau)$ can be expanded into a series as

$$T(M,\tau)=\sum_{i=1}^{\infty}\frac{\psi_i(M,\tau)}{\int_V w(M,\tau)\psi_i^2(M,\tau)\,\mathrm{d}V}\bar{T}_i(\tau) \quad (7)$$

Some mathematical operations fully analogous to these in [1] leads to:

$$\gamma(\tau)\int_V w(M,\tau)\psi_i(M,\tau)\frac{\partial T(M,\tau)}{\partial\tau}\mathrm{d}V+[\mu_i^2(\tau)-\beta(\tau)]\bar{T}_i(\tau)=g_i(\tau) \quad (8)$$

where

$$g_i(\tau)=\int_S k(N,\tau)\varphi(N,\tau)\frac{\psi_i(N,\tau)-[\partial\psi_i(N,\tau)/\partial n]}{A(N,\tau)+B(N,\tau)}\mathrm{d}S+\int_V \psi_i(M,\tau)P(M,\tau)\,\mathrm{d}V. \quad (9)$$

From the relation

$$\frac{\partial}{\partial\tau}[w(M,\tau)\psi_i(M,\tau)T(M,\tau)]=w(M,\tau)\psi_i(M,\tau)\frac{\partial T(M,\tau)}{\partial\tau}+T(M,\tau)\frac{\partial}{\partial\tau}[w(M,\tau)\psi_i(M,\tau)] \quad (10)$$

follows

$$\int_V w(M,\tau)\psi_i(M,\tau)\frac{\partial T(M,\tau)}{\partial\tau}\mathrm{d}V=\frac{\mathrm{d}\bar{T}_i(\tau)}{\partial\tau}-\int_V T(M,\tau)\frac{\partial}{\partial\tau}[w(M,\tau)\psi_i(M,\tau)]\,\mathrm{d}V. \quad (11)$$

Substituting equation (7) in (11) and the result obtained in (8), one gets

$$\frac{\mathrm{d}\bar{T}_i(\tau)}{\mathrm{d}\tau}+\frac{\mu_i^2(\tau)-\beta(\tau)}{\gamma(\tau)}\bar{T}_i(\tau)-\sum_{j=1}^{\infty}\frac{\Omega_{ji}(\tau)\bar{T}_j(\tau)}{\int_V w(M,\tau)\psi_j^2(M,\tau)\,\mathrm{d}V}=\frac{g_i(\tau)}{\gamma(\tau)}\quad i=1,2,\ldots \quad (12)$$

where

$$\Omega_{ji}(\tau)=\int_V \psi_j(M,\tau)\frac{\partial}{\partial\tau}[w(M,\tau)\psi_i(M,\tau)]\,\mathrm{d}V. \quad (13)$$

The functions $\bar{T}_i(\tau)$ being obtained from (12), the desired function $T(M,\tau)$ straight forward determined by the inversion formula (7).

If $w(M,\tau)$, $k(M,\tau)$, $\rho(M,\tau)$, $A(N,\tau)$ and $B(N,\tau)$ were independent of time, i.e. $\psi_i(M,\tau)$ and $\mu_i(\tau)$ were independent of time too, the system (12) would have been a system of uncoupled ordinary, first order, linear differential equations, which are solved in [1].

The method presented here can easily be applied to problems analogous to the ones treated in [3, 4] but with time dependent coefficients. In [5] are presented the solution of the particular case when $\gamma(\tau)=1$, $\beta(\tau)=0$ and $w(M,\tau)$, $k(M,\tau)$, $\rho(M,\tau)$, $A(N,\tau)$ are time independent. In the same paper a method is described for the solution of the system (12) through s-order approximation.

REFERENCES

1. M. D. Mikhailov, General solution of the heat equation in finite regions, *Int. J. Engng Sci.* **10**, 577–591 (1972).
2. Yu. V. Vidin, Heating of a cylinder with an internal heat source and variable heat-transfer coefficient, *J. Engng Phys.* **11**, 166–170 (1966).
3. M. D. Mikhailov, General solutions of the coupled diffusion equations, *Int. J. Engng Sci* **11**, 235–241 (1973).
4. M. D. Mikhailov, General solutions of the diffusion equations coupled at boundary conditions, *Int. J. Heat Mass Transfer* **16**, 2155–2164 (1973).
5. M. N. Özişik and R. L. Murray, On the solution of linear diffusion problems with variable boundary condition parameters, *Trans. Am. Soc. Mech. Engrs* **C**, **96**, 48–51 (1974).